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The Nobel Prizes of 2011

The accelerated universe. On the Nobel Prize in Physics 2011 awarded to Saul Perlmutter, Brian P. Schmidt, and Adam G. Riess*

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Resum. Des de final de la dècada del 1920 hem sabut que les galàxies distants s'allunyen de nosaltres. Les observacions que van conduir a aquesta conclusió van ser principalment les d'Edwin Hubble. La història de l'Univers ha estat de contínua expansió i refredament, i marcada per diversos esdeveniments importants. En un univers dominat per la matèria, és bastant intuïtiu pensar que l'expansió es frenarà o, en altres paraules, que l'Univers s'hauria de desaccelerar. I no obstant això, dos equips, el *Supernova Cosmology Project* i el *High-z Supernova Search Team*, van utilitzar un subconjunt de supernoves del tipus Ia (SNIA) i van arribar al mateix resultat sorprenent: l'Univers s'està accelerant. Però llavors, què està produint l'observada acceleració cosmològica? En aquest article es discuteix el Premi Nobel de Física 2011, atorgat a Saul Perlmutter, Brian P. Schmidt i Adam G. Riess pel descobriment de l'expansió accelerada de l'Univers mitjançant observacions de supernoves distants, i revisa el context cosmològic del descobriment i l'ús de les supernoves com candeles estàndard. Algunes de les conseqüències del descobriment també es presenten.

Paraules clau: cosmologia · supernova · constant cosmològica · energia fosca

Abstract. Since the end of the 1920s we have known that distant galaxies are receding from us. The observations that led to this conclusion were mainly those of Edwin Hubble. The history of the universe has been one of continuous expansion and cooling, marked by several critical events. In a matter-dominated universe, it is quite intuitive that the expansion will eventually slow down; in other words, the universe should decelerate. And yet two teams, the Supernova Cosmology Project and the High-z Supernova Search Team, used a subset of supernova—type Ia (SNIa)—and reached the same surprising result: the universe is accelerating. But what, then, is producing the observed cosmological acceleration? This article discusses the 2011 Nobel Prize for Physics, awarded to Saul Perlmutter, Brian P. Schmidt, and Adam G. Riess for the discovery of the accelerating expansion of the universe through observations of distant supernovae, and reviews the cosmological context of the discovery and the use of supernovae as standard candles. Some of the consequences of the discovery are presented as well.

Keywords: cosmology · supernova · cosmological constant · dark energy

Introduction

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae." (Fig. 1)

What today we call cosmology, i.e., the study of the cosmos, has a very long history. In ancients times it was mixed with religion, philosophy, etc. Today cosmology is its own field but the need of humanity to understand the universe has remained the same. The present article does not include a description of the evolution of ideas in cosmology and the many people that have contributed to achieving progress in the un-

Fig. 1. From left to right Saul Perlmutter, Brian P. Schmidt, and Adam G. Riess © The Nobel Foundation. Photos: Ulla Montan.

* Based on the lecture given by the author at the Institute for Catalan Studies, Barcelona, on 16 December 2011, and at the Octubre Centre of Contemporary Culture, Valencia, on 11 January 2012 for the Nobel Prize Cycle.

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derstanding of the cosmos; rather, it concentrates on one of the more recent discoveries: the fact that the expansion of the universe is accelerating. Besides the topics explained in the lectures at the Institute for Catalan Studies and Octubre Centre of Contemporary Culture, this article is aimed at a slightly more advanced level and includes both related references and a bibliography for the reader interested in finding out more about the subject. In the Introduction, I discuss the context in which the acceleration of the universe arises, namely, universal expansion.

Since the end of the 1920s we have known that distant galaxies are receding from us. The observations that led to this conclusion were mainly those of Edwin Hubble [11]. We know that the universe expands because when a distant galaxy long ago sent a light signal the universe was smaller than when that signal is received on the Earth. Thus, in the expansion, the wavelength of light was stretched by the same factor as the scale of the universe. We receive light from astronomical objects with the spectrum lines red-shifted. Thus, at the cosmological scale, we have:

$$\frac{\lambda_r}{\lambda_e} = \frac{a_0}{a} = 1 + z \tag{1}$$

Here λ_e is the wavelength at the moment of emission, λ_r is the wavelength at the moment of reception (now), a is the scale at the moment of emission, and a_0 is the scale now. Finally, z is defined as the red-shift. It is quite usual to refer to the red-shift of distant galaxies, instead of the physical distance. For example, $z = 1$ corresponds to about 8000 Mly, where Mly stands for 10^6 light years. At red-shift $z = 1$, the dimensions of the universe were half of what they are today.

Before we continue, several comments are in order. First, there is nothing special about our position on Earth from where we measure all distant objects to recede from us. The Copernican principle states that we do not occupy a privileged place in the cosmos. Let us consider a triangle that has a galaxy at each of its three vertices. After a cosmological time has elapsed, the triangle formed by the three galaxies is larger but it is similar (same shape, i.e., same angles) to the first triangle. From the point of view of any of the vertices, an observer sees the other two galaxies receding radially. This is in agreement with the Copernican principle, which is an extension of the Copernican idea that the Earth is not the center of the solar system.

Second, let us assume that the growing rate of the triangle is constant. As we will see, this is not exact, but it is a very good first approximation. The assumption of constant expansion implies Hubble's law, which describes the linear relation between distance and red-shift. However, there are several ways to define cosmological distances. Here we use the so-called luminosity distance d_L,

$$d_L = \left(\frac{F}{4\pi L}\right)^{1/2} \tag{2}$$

where F is the flux received and L is the luminosity of the astrophysical object. According to Hubble's law:

$$cz = H_0 d_L \tag{3}$$

The proportionality constant H_0 is the Hubble constant, i.e., the expansion rate, and c is the speed of light. The expansion of the universe is probably the most fundamental cosmological property. If we go backward in time, distances among objects become increasingly smaller while densities and temperature steadily become higher. In the very early universe, all atoms were ionized, and the content of the universe was a primordial plasma.

The history of the universe has been one of continuous expansion and cooling, marked by several critical events. For example, within the first few minutes, the fusion of protons and neutron resulted in the formation of light elements. Later, at 450,000 years, the decoupling of photons occurred. We are able to observe these relic particles and from their properties we can extract valuable information about our universe.

The acceleration of the universe

In a matter-dominated universe, it is quite intuitive that the expansion will eventually slow down; in other words, the universe should decelerate. It is analogous to what happens when we throw an object upwards in the gravitational field of the Earth. The object loses velocity because of the attraction of the Earth. In the universe, attraction is exerted by all masses among themselves, implying an eventual deceleration.

To express this formulaically, we write the evolution equations for the scale parameter a:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) \tag{5}$$

where G_N is Newton's gravitational constant, and ρ and p the energy density and pressure of the fluid content of the universe, respectively. The equations include the first temporal derivative of the scale factor, $\dot{a} = da/dt$, as well as the second derivative $\ddot{a} = d^2a/dt^2$. These equations correspond to the particular case of a flat universe, which is an excellent approximation.

Equations (4) and (5) are directly deduced from the Einstein equations of general relativity [5], assuming a homogeneous and isotropic universe. The first steps of applying general relativity to cosmology were done by Einstein [6] and were followed by important contributions from Friedmann [8] and Lemaître [13]. In a matter-dominated universe $p = 0$ and, since $\rho > 0$, the minus sign in (5) shows that $\ddot{a} < 0$. Therefore, we conclude that a matter dominated universe is decelerating.

In physics, as in all branches of science, experiments must be performed without prejudice. In principle, the universe could decelerate, $\ddot{a} < 0$, or accelerate $\ddot{a} > 0$, or $\ddot{a} = 0$. As stated above, Hubble's law (3) has to be regarded as a first approximation, valid for a small z, i.e., objects not very far away. In order to measure the acceleration or deceleration, we must first measure the dependence between z and d_L beyond the linear approximation expressed by Hubble's law. We should make more precise measurements as well as obtain additional data for more distant objects. To measure the red-shift z is not especially difficult, the hard part is to measure the distance.

We can use an example to understand why distance measurements are difficult. Imagine we are in a dark room except for two candles. We measure the light coming from both and find it is equal. Can we conclude that the candles are at the same distance? Of course not, because one candle might be more luminous than the other but be farther away so that there is a kind of compensation and the light we receive is the same. This simple example shows that if we knew whether the candles were identical then we could draw our conclusions without any problem. In the language of science, these identical candles are known as standard candles. When we are sure that we measure light coming from standard candles we can indeed conclude that, if the light we receive from the two is the same, the candles are at the same distance; if the light from one is one fourth of that from the other, it is at twice the distance, etc.

In astronomy, the crucial point is to find standard candles, i.e., astronomical objects with the same intrinsic luminosity. Supernovae as standard candles were proposed many years ago [2]. However, while they are extremely bright objects, outshining their own host galaxy for a period of time, their intrinsic luminosities are very different and they are not standard candles. Physicists have found a solution to this problem by identifying a subset of supernova, called type Ia (SNIa), which are remarkably similar [4]. These supernovae were identified based on several spectral features, specifically, the presence of lines of ionized silicon but no hydrogen lines. Underlying this similarity is a common origin. Indeed, it is believed that the origin might have been the explosion of a white dwarf in a binary system, where the companion would have become a red giant [7], and that accretion, i.e., matter attracted and integrated by the white dwarf, would have occurred. White dwarfs are end states of stars that compensate the gravitational attraction by the pressure of degenerate electrons. However, when the mass increases and reaches the so-called Chandrasekhar limit (1.4 solar masses), the electron pressure is not able to stop the gravitational collapse. Consequently, nuclear reactions start, which can lead to a violent explosion. The fact that all white dwarfs becoming SNIa explode when reaching the Chandrasekhar mass is the reason behind the similarity in the intrinsic luminosities of this class of supernovae.

SNIa are the standard candles that have been used to infer the measure of distances to the host galaxies where supernovae are detected [15]. Actually, there is a small dispersion in intrinsic luminosities: some of them are brighter at peak and have a longer duration and some are dimmer and have a shorter duration. This correlation is such that one can rescale all luminosities in a single profile, with the end result that the dispersion is even smaller [17] (Fig. 2). The use of supernovae as standard candles allows us to determine the relative distances among them. One introduces into the sample nearby supernovae whose absolute distances can be deduced by other means, such as parallaxes. It then follows that the absolute distance to of all such objects can then be determined.

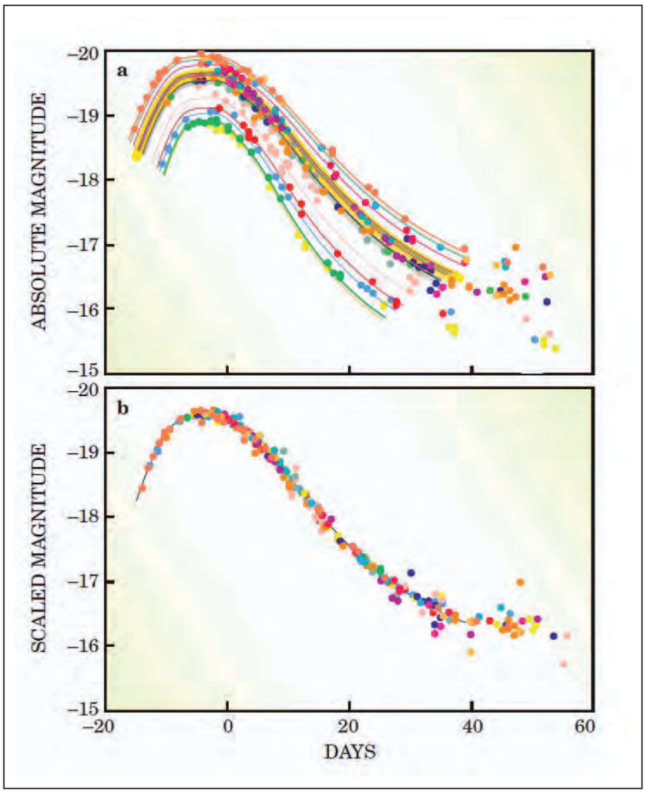

Fig. 2. Light curves of low red-shift type Ia supernovae measured by [9] and [10]. The upper figure shows the absolute magnitude (a measure of intrinsic luminosity) as a function of time. In the lower figure, the same light curves after the rescaling explained in the text. (From [14].)

We can now try to find deviations from Hubble's law (3). Working with the cosmological equations one, we achieve the relation:

$$d_L = \frac{c}{H_0}\left(z + \frac{1}{2}(1 - q_0)z^2 + \ldots\right) \tag{6}$$

Here we identify the first term in z as Hubble's law, so that the term z^2 amounts to a correction that yields information about acceleration. Indeed, the deceleration parameter

$$q_0 = -\frac{\ddot{a}}{aH_0^2} \tag{7}$$

contains the second derivative of the scale parameter $\ddot{a}$. In (10), the dots stand for the terms in z^3. Of course, at $z \sim 1$ a more general formula is needed. This formula is introduced in the next section.

Two teams used SNIa to plot the red-shift of these objects versus distance: the Supernova Cosmology Project [16] and the High-z Supernova Search Team (Fig. 3) [18]. Both reached the same surprising result: the universe is accelerating. Their result can be expressed as a determination of q_0 in (7). Due to the conventional minus sign in the definition (7), a positive value $q_0 > 0$ indicates a decelerating universe and a negative value

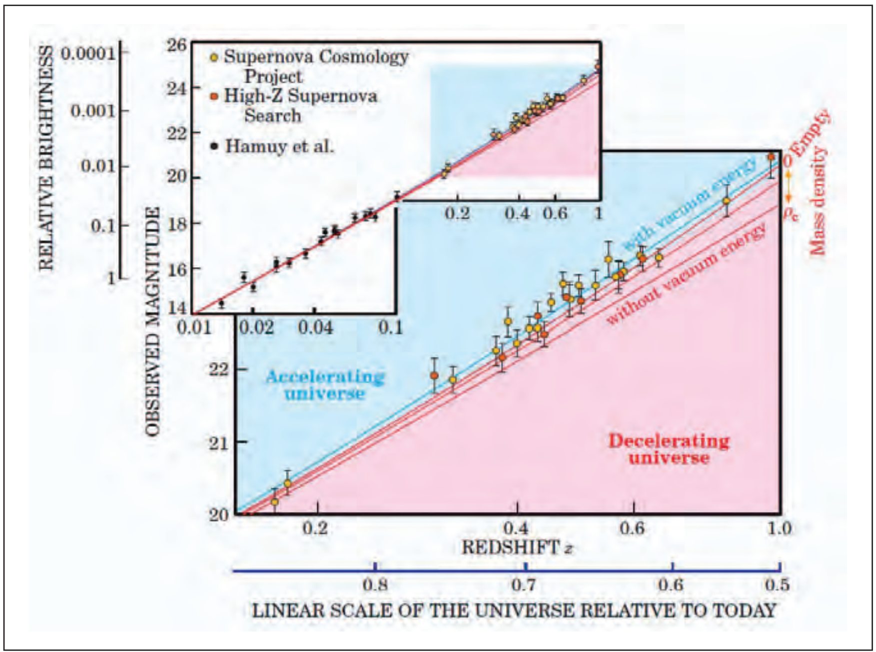

Fig. 3. Observed magnitude versus red-shift. The part corresponding to the most distant supernovae is enlarged to more clearly depict the fit. The pink part of the figure corresponds to a decelerating universe ,and the blue part to an accelerating universe. The best fit is for an accelerating universe with about 0.24 ρ_c in matter density and 0.76 ρ_c in cosmological constant density. (From [14]).

$q_0 < 0$ means the universe is speeding up. The two teams obtained a negative value in 1998, which led them to infer that the universe is accelerating. The present experimental value that can be deduced from supernova data is $q_0 = -0.7 \pm 0.1$.

I should point out that these measurements have been possible because of technical advances in astronomy: telescopes, CCD detectors, etc. Also, an alert program was crucial because the light curve has to be measured before peak brightness (and of course after peak brightness).

Consequences

As equations (4) and (5) make clear, the expansion features are linked to the energy content of the universe. I already noted that matter in the universe produces deceleration. But what, then, is producing the observed cosmological acceleration?

Actually, in the beginning of the development of general relativity, a term that could contribute to the gravitational equations describing cosmological acceleration was considered. That term contained a single free parameter, Λ, and it is known as a cosmological constant. For example, it was considered by Einstein himself in order to obtain a static universe, but he abandoned the idea after the discovery of the expansion of the universe. The possible presence of a cosmological constant should be definitely considered because of the cosmological observations using supernovae. The cosmological constant is equivalent to a universal component entering the right-hand side of (4) and (5). This component has an energy density ρ_Λ and a negative pressure $p_\Lambda = -\rho_\Lambda$. We easily see in (5) that it is a contribution to positive values for $\ddot{a}$, namely, to acceleration.

Let us consider a universe with matter density ρ_M and cosmological constant density ρ_Λ. It is convenient to work with the normalized quantities

$$\Omega_M = \frac{\rho_M}{\rho_c} \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \tag{8}$$

where ρ_c is the critical density, which is the density leading to a flat universe, given by

$$\rho_c = \frac{3H_0^2}{8\pi G_N} \tag{9}$$

Let us write a formula relating distances and the parameters of this universe

$$d_L = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3+\Omega_\Lambda}} \tag{10}$$

[A second-order expansion of this formula in z leads to (10)]. The supernova data relate d_L to z so they can infer allowed and not-allowed values for Ω_M and Ω_Λ.

Figure 4 shows a plane that is the parameter space of a universe with matter and a cosmological constant. The supernova data are compatible only with a region in the plane; in Fig. 4 the region is represented by the blue band. It can be seen that the data do not point to a single point in the plane because there are experimental uncertainties. Moreover, the region consistent with supernova data is a band, indicating that there is degeneracy. In the plot, the band coming from the cosmic microwave background is shown in orange, and the band corresponding to the so-called baryon acoustic oscillations in green. The bands are again due to the fact that each of these observations have degeneracy. By considering all data at the same time we can remove the degeneracy. In fact, all data can be accommodated by adopting the values $\Omega_M \approx 0.26$ and $\Omega_\Lambda \approx 0.74$. This simple model is sometimes called the consistency model. (For an alternative approach to display results see [3].)

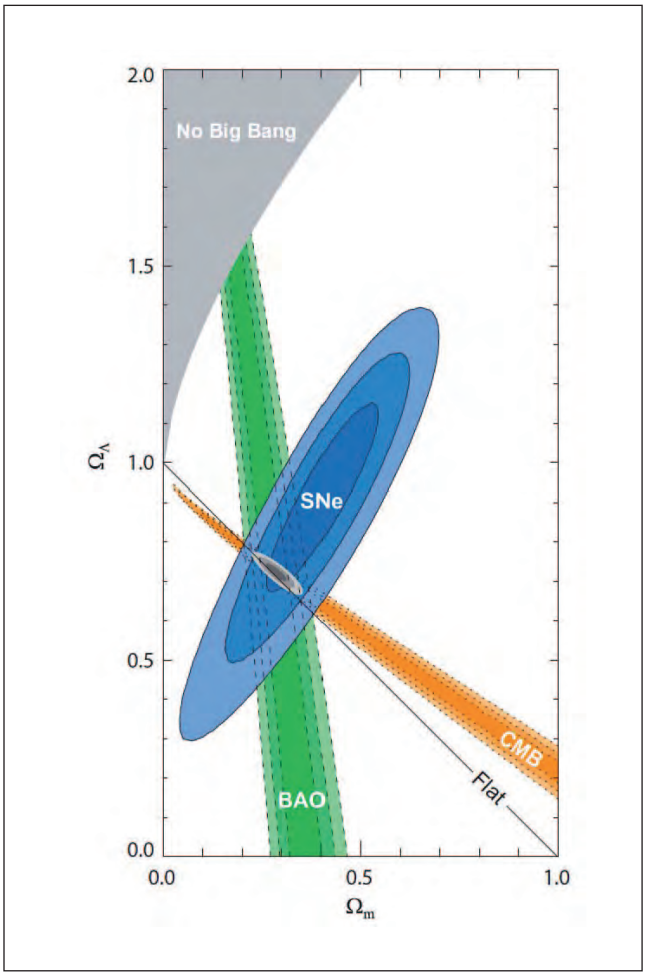

Fig. 4. Parameter space of a simple model with matter and the cosmological constant. In blue, the region determined by supernovae data. In orange, the region consistent with cosmic microwave background measurements is shown; in green, the part delimited by galaxy cluster inventories. The confidence level contours of 68.3 %, 95.4 %, and 99.7 % are shown as different color densities. The best fit corresponds to an accelerating universe with about 0.24 ρ_c in matter density and 0.76 ρ_c in cosmological constant density (w = –1). (From [12]).

The cosmological model with matter and a cosmological constant is consistent with the data. Is this the end of the story? We believe it is not, for the following reasons. The value needed for the cosmological constant to fit the data is extremely small in the following sense. According to quantum mechanics, the vacuum fluctuates constantly. There is a constant creation of matter-antimatter pairs that are immediately annihilated. Such fluctuations are allowed by the Heisenberg principle. The energy density associated with these fluctuations is many orders of magnitude above the observational value. The conclusion is that we do not understand this small value. Also, the contributions of matter density and cosmological density turn out to be not very different from each other; they are similar up to a factor of two or so. This similarity happens only within a very short time interval, in cosmological time scales. This is also not understood.

These problems have originated a large amount of theoretical work searching for alternatives to the cosmological constant. For example, it might be that the origin of the cosmic acceleration is the existence of a fluid with exotic properties, actually with a negative pressure. Let us call ρ_{DE} and p_{DE} the energy density and pressure, respectively, and define the ratio

$$w = \frac{p_{DE}}{\rho_{DE}} \tag{11}$$

As noted above, w = –1 corresponds to a cosmological constant. In general, models of dark energy have w = –1 and are dependent on time. Models for such fluids have been constructed, but we do not have yet a truly convincing model. Another possibility is that gravity is modified when considering large scales. Such modifications are constrained by the observational fact that in shifting to smaller scales—for example, solar system scales—gravity works perfectly well, without any need to introduce modifications. Again, no convincing model is in sight.

Improvement in the cosmological measurements should give us clues for further progress. For example, Fig. 5 shows a plot that determines w in (11), assumed to be constant, but not necessarily equal to –1. In the plot, there are the same three observations as in Fig. 4. We see that the data are consistent with a value w = –1, i.e., with a pure cosmological constant. However, it remains to be determined whether more refined data in the future select a value $w \neq -1$ or continue to be consistent with w = –1. Apart from supernova data, it is believed that measurements of galaxy cluster abundances, weak gravitational lensing observations, and more precise determination of the properties of baryon acoustic oscillations may lead to progress in understanding the origin of the acceleration of the universe.

As a final remark, Fig. 6 is a chart showing the weight of the different components in the universe according to our present measurements.

Dominating the energy budget is the presence of dark energy, which has a relative importance of 74 %. The remainder

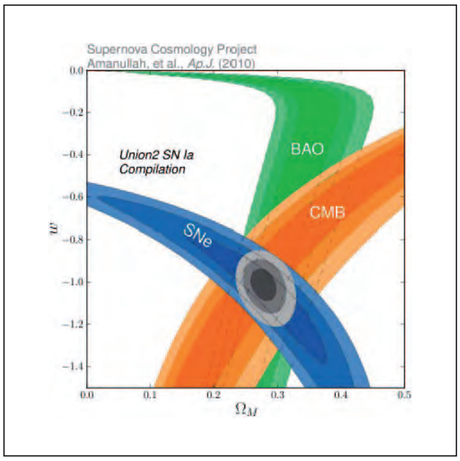

Fig. 5. Parameter space of a simple model with matter and a dark energy component with constant w [see equation (11)]. In blue, the region determined by supernovae data is shown; in orange, the region consistent with cosmic microwave background measurements; in green, the part delimited by the galaxy cluster inventories. Confidence level contours of 68.3 %, 95.4 %, and 99.7 % are shown. The best fit is consistent with a cosmological component w = –1. (From [1]). Reproduced by permission of the AAS.

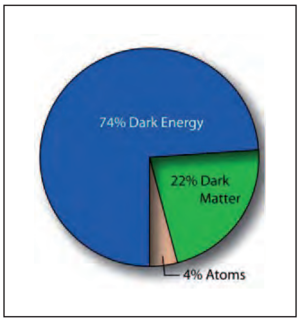

Fig. 6. Only 4 % of the mass of the universe is made up of ordinary matter.

has a weight of 4 % in normal atoms, and 22 % in dark matter. Dark matter should have properties very different from normal matter, and we know it exists because of its gravitational action. The fact that the matter comprising human beings, the Earth, and the sun is a mere 4 % is sometimes called the second Copernican revolution. Indeed, we are not at the center of the universe; rather, the bulk of the universe is made up of components that differ from those we are made of.

Additional bibliography

1. *The Nobel Prize webpage* contains the prize announcement, press release, popular information, and advanced information. The award ceremony video and speeches are also available. In addition, there is a biography for each of the three laureates.
   - http://www.nobelprize.org/nobelprizes/physics/laureates/2011/

2. The websites of the two collaborations, *Supernova Cosmology Project* and *High-z Supernova Search Team*, offer popular science articles as well as more advanced articles, along with many images and graphs.
   - http://supernova.lbl.gov/
   - http://www.cfa.harvard.edu/supernova//HighZ.html

3. *Popular science reviews*
   - Riess AG, Turner MS (2004) From Slowdown to Speed-up. Scientific American Magazine, February
   - Translation into Spanish in Revista Investigación y Ciencia, April 2004
   - http://www.eso.org/public/

4. *Interview* with one of the Nobel Prize winners
   - http://www.scientificamerican.com/article.cfm?id=discovering-a-dark-universe

5. *Wikipedia article*
   - http://en.wikipedia.org/wiki/Dark energy

6. *Related webpages* to the 2011 Nobel Prize in Physics
   - http://www.scientificamerican.com/article.cfm?id=supernovae-backeinsteins
   - http://blogs.scientificamerican.com/cocktail-party-physics/2011/10/04/nobeldreams-2011-physics-prize-honors-accelerating-universe/
   - http://www.scientificamerican.com/article.cfm?id=the-2011-nobel-prize-in-prize-physics

7. *Elementary textbook on cosmology*
   - Andrew Liddle (2003) An Introduction to Modern Cosmology. Wiley, Chichester, England

8. *Elementary reviews* on different aspects related to the 2011 Nobel Prize in Physics
   - Perlmutter S (2003) Supernovae, Dark Energy, and the Accelerating Universe. Physics Today 53-59 [http://supernova.lbl.gov/PhysicsTodayArticle.pdf]
   - Kirshner RP (2003) Throwing Light on Dark Energy. Science 300:1914-1918
   - Mazzali PA, Röpke FK, Benetti S, Hillebrandt W (2007) A Common Explosion Mechanism for Type Ia Supernovae. Science 315:825-828

9. *Advanced reviews on the acceleration of the universe and dark energy*
   - Caldwell RR, Kamionkowski M (2009) The Physics of Cosmic Acceleration. Ann Rev Nucl Part Sci 59:397 [http://arXiv.org/pdf/0903.0866]
   - Copeland EJ, Sami M, Tsujikawa S (2006) Dynamics of dark energy. Int J Mod Phys D15:1753 [http://arXiv.org/pdf/hep-th/0603057]
   - Frieman J, Turner M, Huterer D (2008) Dark Energy and the Accelerating Universe. Ann Rev Astron Astrophys 46:385 (2008) [http://arXiv.org/pdf/0803.0982]
   - Carroll SM (2001) Dark Energy and the Preposterous Universe [http://arXiv.org/pdf/astro-ph/0107571]
   - Peebles PJE, Ratra B (2003) The cosmological constant and dark energy. Rev Mod Phys 75:559 [http://arXiv.org/pdf/astro-ph/0207347]

10. *Advanced reviews on the cosmological constant*
    - Weinberg S (1989) Rev Mod Phys 61:1-23
    - Carroll SM (2001) Living Rev Rel 3:1[http://relativity.livingreviews.org/Articles/lrr-2001-1/]

References

1. R Amanullah, et al. (2010) Spectra and Light Curves of Six Type Ia Supernovae at 0.511 < z < 1.12 and the Union2 Compilation. Ap J 716:712-738
2. Baade W (1938) The Absolute Photographic Magnitude of Supernovae. Ap J 88:285-304
3. Bahcall NA, Ostriker JP, Perlmutter S, Steinhardt PJ (1999) The Cosmic triangle: Assessing the state of the universe. Science 284:14811488
4. Branch D, Tammann GA (1992) Type Ia supernovae as standard candles. Ann Rev Astron Astrophys 30:359-389 [and references therein]

5. Einstein A (1915) Die Feldgleichungen der Gravitation. Sitzungsber Preuss Akad Wiss, Berlin, pp. 844-847
6. Einstein A (1917) Kosmologische Betrachtungen zur allgemeinen Relativit¨atstheorie. Sitzungsber Preuss Akad Wiss, Berlin pp. 844-847
7. Fowler WA, Hoyle F (1960) Nucleosynthesis in supernovae. Astrophys J 132:565-590
8. Friedmann A (1924) Uber die Moeglichkeiten einer Welt mit konstanter negativer Krümmung des Raumes. Zeitschrift für Physik 21:326-332
9. Hamuy M, et al. (1993) The 1990 Calan/Tololo supernova search. Astrophys J 106:2392-2407
10. Hamuy M, et al. (1995) A Hubble diagram of distant type IA supernovae. Astrophys J 109:1-13
11. Hubble E (1929) A relation between distance and radial velocity among extra–galactic nebulae. Proc Nat Acad Sci 15:168-173
12. Kowalski M, et al. (2008) [Supernova Cosmology Project Collaboration] Improved Cosmological Constraints from New, Old and Combined Supernova Datasets. Astrophys J 686:749-778 [http://arxiv.org/abs/0804.4142]
13. Lemaître G (1927) Un univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extragalactiques. Annales de la Société Scientifique de Bruxelles A47:4959
14. Perlmutter S (2003) Supernovae, Dark Energy, and the Accelerating Universe. Physics Today, April, pp. 53-59
15. Perlmutter S, Schmidt BP (2003) Measuring cosmology with supernovae. Lect Notes Phys 598:195-217 [http://arxiv.org/abs/astro-ph/0303428]
16. Perlmutter S, et al. (1999) Measurement of Ω and Λ from 42 high-redshift supernovae. Astrophys J 517:565-586
17. Phillips MM (1993) The absolute magnitude of Type Ia supernovae. Astrophys J 413:L105-L108
18. Riess AG, et al. (1998) Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron J 116:10091038